

## Skyrmions from a Born-Infeld Action

B. Dion, L. Marleau and G. Simon  
*Département de Physique, Université Laval*  
*Québec, Canada, G1K 7P4*  
(July 1994)

### Abstract

We consider a geometrically motivated Skyrme model based on a general covariant kinetic term proposed originally by Born and Infeld. We introduce this new term by generalizing the Born-Infeld action to a non-abelian  $SU(2)$  gauge theory and by using the hidden gauge symmetry formalism. The static properties of the Skyrmion are then analyzed and compared with other Skyrme-like models.

PACS numbers: 11.30.Na, 12.39.Dc, 12.60.Cn.

## I. INTRODUCTION

For many years, the non-linear  $\sigma$ -model has been known to provide an approximate description of hadronic physics in the low-energy limit. Moreover, its finite-energy configuration space exhibits a non-trivial topological structure which allows the existence of static and finite field configurations other than the trivial vacuum. Unfortunately, the  $\sigma$ -model Lagrangian being of second order in field derivatives, these so-called soliton configurations are not energetically stable in ordinary three-dimensional space.

In a celebrated series of articles, Skyrme [1] proposed to modify the Lagrangian in order to include a quartic term that prevents the model from being renormalizable. His model provides the ground state properties of the nucleon to within about 30% accuracy. The new term insures dynamically stable solitons (or Skyrmions), which Skyrme suggested to identify with baryons. Little physical significance could be attached to this term however, except for the fact that it could arise as a manifestation of higher-spin meson exchanges.

In recent years, renewed interest has been paid to the Skyrme model in the context of weak interactions as well as in low-energy QCD with the introduction the hidden gauge symmetry formalism. It was then understood that the Skyrme quartic term could be formally interpreted as the kinetic contribution of a vector gauge field in the limit where the mass of this field becomes infinite. Indeed, various authors [2] have shown that the nonlinear  $SU(2)$   $\sigma$ -model with spontaneously broken chiral symmetry was equivalent to a linear model based on the group  $SU(2)_L \otimes SU(2)_R \otimes SU(2)_V$  where  $SU(2)_V$  is assumed to be a hidden symmetry. This new vector field becomes dynamical when we add a kinetic term of the form  $Tr[F_{\mu\nu}F^{\mu\nu}]$  to the Lagrangian. When the mass of the vector field is taken to be infinite, the latter decouples and we find [3]  $F_{\mu\nu} \rightarrow [L_\mu, L_\nu]$  where  $L_\mu = U^\dagger \partial_\mu U$  and  $U$  is the pion field. We then recover the Skyrme Lagrangian.

On the other hand, the requirement of gauge invariance is not sufficient to fix a precise form of the gauge term. It could in principle include – in addition to a usual gauge kinetic term – terms with higher orders, or even non-polynomial functionals, of the field derivatives. Of course, in these cases, the resulting theory loses explicit renormalizability but it is hoped that there exists some mechanism of cancellation of divergences that may render physical results finite. In a paper dating back to 1934, Born and Infeld [4] proposed a general covariant action function, which has the attractive features of arising directly from the metric. In this work, we will consider a generalization of the Born-Infeld action to describe chiral dynamics. Indeed, there exist a number of analogies between chiral dynamics and general relativity [5] and the idea of a direct connection is very appealing. They are both intrinsically non-linear and are characterized by a non-trivial geometrical structure. Moreover, the compactification of space from  $R^3$  to  $S^3$ - due to the necessary boundary conditions - implies that the soliton field effectively “sees” a closed physical space  $S^3$ . Thus the analogy with a closed Einstein universe with hadronic size radius.

The Born-Infeld action was originally built as a revival of the old model of the electromagnetic origin of mass. It has the property of insuring absolute finiteness of energy and of reducing to the usual  $Tr[F_{\mu\nu}F^{\mu\nu}]$  Maxwell form in the low-energy limit. Another interesting feature of this theory is the non-linear nature of the field equations. The latter are distinguished by a mass scale  $M$  which suggests an analogy with the notion of effective Lagrangians where we have integrated over the heavy particles’ degrees of freedom. Whereas

Born and Infeld introduced this action in the context of  $U(1)_{EM}$ , we intend in this work to generalize it to  $SU(2)$  chiral dynamics, using the hidden gauge symmetry formalism mentioned above. Our analysis and computations mainly deal with hadron physics which is the best prototype available (since physical quantities are easier to identify) but the idea and most conclusions could easily be transposed to other contexts, e.g. weak Skyrmions. The Skyrmions we described are themselves characterized by a length scale  $1/M$ .

The remainder of this paper is divided in four sections. First, we present the concept and principal features of the Born-Infeld action and introduce a proper choice of generalization to  $SU(2)$ . Then, we build an effective chiral Lagrangian using the HGS formalism. We finally proceed to derive and discuss the physical properties of resulting Skyrmions in the context of hadron physics.

## II. THE BORN-INFELD ACTION

The original purpose of Born and Infeld [4] was to introduce a new electrodynamic field theory in which the self-energy of a point charge would be finite. They proposed to apply the principle of finiteness - which says that all physical quantities should be finite everywhere - to the electromagnetic field by postulating an absolute value  $b$  such that  $F_{\mu\nu} \rightarrow b$  in the high-energy limit. The  $b$  constant has dimensions  $[M]^2$  where  $M$  can be interpreted as a scale parameter. The new kinetic term they derived bears a striking resemblance to the relativistic expression of the kinetic energy of a particle, with the velocities  $v$  and  $c$  replaced by the field strengths  $F_{\mu\nu}$  and  $b$ .

Apart from the finiteness of energy, our interest in the Born-Infeld Lagrangian is enhanced by the following properties:

(i) An intrinsic length scale reminiscent of an effective Lagrangian with the heavy particles' degrees of freedom integrated over.

(ii) The Lagrangian is constructed out of the two Maxwell invariants of only  $F_{\mu\nu}F^{\mu\nu}$  and  $F_{\mu\nu}^*F^{\mu\nu}$ , not derivatives such as  $[D^\lambda F^{\mu\nu}]^2$ .

(iii) It is intrinsically non-linear but still, it can be quantized [6] and is the only causal spin-1 theory [7] aside from the gauge Lagrangian,  $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ .

(v) Most of all, it is geometric by nature. It is one of the simplest non-polynomial Lagrangians that is invariant under the general coordinate transformations.

The Born-Infeld action is based on the invariant measure written as

$$\delta \int \mathcal{L} d\tau = 0, \quad (d\tau = dx^0 dx^1 dx^2 dx^3)$$

where the metric is considered to be represented by a general non-symmetrical tensor  $a_{\mu\nu}$ . The symmetrical part is the usual space-time metric  $g_{\mu\nu}$  and the anti-symmetrical part is identified with the electromagnetic field  $F_{\mu\nu}$ . The general covariant action which reproduces the Maxwell action in the limit of flat space-time and lowest order in the field strength is then

$$\mathcal{L}_{BI} = b^2 \left( \sqrt{-|g_{\mu\nu} + F_{\mu\nu}|} - \sqrt{-|g_{\mu\nu}|} \right) \quad (1)$$

where  $|g_{\mu\nu}| = \det g_{\mu\nu}$ .

Assuming flat space-time (i.e.  $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ ), the last expression can be written in the form

$$\mathcal{L}_{BI} = b^2 \left( \sqrt{1 + K - G^2} - 1 \right) \quad (2)$$

where

$$K = \frac{1}{b^2} F_{\mu\nu} F^{\mu\nu} \quad (3)$$

$$G = \frac{1}{2b^2} F_{\mu\nu} F^{*\mu\nu} \quad (4)$$

and  $F^{*\mu\nu}$  is the dual field of  $F^{\mu\nu}$ . In the weak field approximation ( $|F| \gg |G^2|$ ) or in the case of static configuration ( $G = 0$ ) we can write

$$\mathcal{L}_{BI} = b^2 \left( \sqrt{1 + \frac{1}{b^2} \text{Tr} [F_{\mu\nu} F^{\mu\nu}]} - 1 \right) \quad (5)$$

In the low-energy limit,  $F_{\mu\nu} \rightarrow 0$  and one recovers the familiar Maxwell form which in our notation is

$$\mathcal{L}_{BI} \rightarrow \frac{1}{2} \text{Tr} [F_{\mu\nu} F^{\mu\nu}] \quad (6)$$

It is then easy to derive the conservation laws of energy-momentum and to describe the finite-energy electron by a static solution with spherical symmetry and finite spatial extension. One finds that the  $\left| \frac{1}{b^2} \text{Tr} [F_{\mu\nu} F^{\mu\nu}] \right|$  term to tend asymptotically to 1 in the high energy limit.

In order to describe  $SU(2)$  hadron physics, we need to generalize the Born-Infeld idea to non-abelian  $SU(2)$  gauge theory. This is done by introducing a non-abelian metric:

$$g_{\mu\nu} + F_{\mu\nu} \rightarrow \mathbf{a}_{\mu\nu} \equiv g_{\mu\nu} \mathbf{1} + F_{\mu\nu}^a \tau^a$$

where  $\tau^a$  are the Pauli matrices. The Lorentz structure of the metric is similar to  $U(1)$  case so the BI action will again involve the determinant of the metric (over space-time indices)

$$|\mathbf{a}_{\mu\nu}| = \frac{1}{4!} \epsilon^{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\gamma\delta} \mathbf{a}_{\mu\alpha} \mathbf{a}_{\nu\beta} \mathbf{a}_{\rho\gamma} \mathbf{a}_{\sigma\delta} \quad (7)$$

But the Lagrangian must be an  $SU(2)$  singlet which involves an operation on the  $SU(2)$  component of  $|\mathbf{a}_{\mu\nu}|$ . The simplest extension is given by taking the trace [8]. The Lagrangian then takes the form:

$$\mathcal{L}_{BI}^{SU(2)} = \frac{b^2}{2} \left( \sqrt{-\text{Tr} |\mathbf{a}_{\mu\nu}|} - \sqrt{-\text{Tr} [g_{\mu\nu} \mathbf{1}]} \right) = b^2 \left( \sqrt{1 + K - \frac{1}{3}(G^2 + 2G_S^2)} - 1 \right) \quad (8)$$

where

$$K = \frac{1}{b^2} F_{\mu\nu}^a F^{a\mu\nu} \quad (9)$$

$$G = \frac{1}{2b^2} F_{\mu\nu}^a F^{*a\mu\nu} \quad (10)$$

$$G_S = \frac{1}{4b^2} [F_{\mu\nu}^a F^{*b\mu\nu} + F_{\mu\nu}^b F^{*a\mu\nu}]. \quad (11)$$

However, in general, any non-polynomial Lagrangian is expected to be a function of nine gauge invariants [9]. Some of these invariants will not contribute because of the symmetries in the definition of the determinant in (7) but otherwise one can think of a very special case where the operator  $\mathcal{O}$  is such that

$$\mathcal{O}(|\mathbf{a}_{\mu\nu}|) = -(1 + \frac{1}{2}K)^2, \quad \mathcal{O}(|g_{\mu\nu}\mathbf{1}|) = -1$$

and this leads to the usual gauge kinetic term  $\mathcal{L}_{BI}^{\mathcal{O}} = \frac{1}{2} \text{Tr} F_{\mu\nu}^a F^{a\mu\nu}$  which has its origin in the metric but no longer constrains the fields to be finite.

In this work, we will only consider the simplest form of non-abelian generalization of the Born-Infeld action in (8) motivated by its constraint of finiteness on the fields.

We now give a brief review of the concepts behind the hidden gauge symmetry formulation.

### III. HIDDEN GAUGE SYMMETRY

The HGS formalism is based on the manifold  $SU(2)_L \otimes SU(2)_R \otimes SU(2)_V$  where  $SU(2)_V$  is gauged. The most general Lagrangian involving only two field derivatives is expressed as

$$\mathcal{L}_2 = -\frac{f_\pi^2}{4} \text{Tr} (L^\dagger D_\mu L - R^\dagger D_\mu R)^2 - ag^2 \frac{f_\pi^2}{4} \text{Tr} (L^\dagger D_\mu L + R^\dagger D_\mu R)^2 \quad (12)$$

where  $L(x) \in SU(2)_L$  and  $R(x) \in SU(2)_R$ . The covariant derivative  $D_\mu$  reads

$$D_\mu = \partial_\mu - igV_\mu^k \cdot \frac{\tau^k}{2} \quad (13)$$

where  $V_\mu$  stands for the hidden gauge field. The first part of equation is equivalent to the gauged non-linear  $\sigma$ -model since it can be brought to the form

$$\mathcal{L}_{NL\sigma} = -\frac{f_\pi^2}{4} \text{Tr} (D_\mu U^\dagger D^\mu U) \quad (14)$$

where the classical Euler-Lagrange equation corresponding to (12) has been used for the auxiliary field

$$V_\mu = \frac{1}{2ig} (L^\dagger \partial_\mu L + R^\dagger \partial_\mu R). \quad (15)$$

The hidden gauge field becomes dynamical when we add a kinetic piece to the Lagrangian.. Here, we use the Born-Infeld form and write

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_{BI}^{SU(2)} \quad (16)$$

The vector boson  $V$  acquires its mass from the same mechanism as the standard gauge bosons with result,  $m_V^2 = ag^2 f_\pi^2$ . In the  $m_V^2 \rightarrow \infty$  limit, we can show that the field strength  $F_{\mu\nu} \rightarrow \frac{1}{4e} \text{Tr} [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]$ . Writing  $L_\mu = U^\dagger \partial_\mu U$ , we now have

$$\mathcal{L}_{tot} = -\frac{f_\pi^2}{4} \text{Tr}(L_\mu L^\mu) + b^2 \left( \sqrt{1 + \frac{1}{16e^2 b^2} \text{Tr}[L_\mu, L_\nu]^2} - 1 \right) \quad (17)$$

This is the Lagrangian form we will be using in the following section to derive the Skymion phenomenology when applied to hadrons. Note that here, we could have written down this Lagrangian right from the beginning without any reference to HGS. The anti-symmetrical part of the metric would be due to the scalar self-interaction. The idea that the Lagrangian (17) may come from HGS is however attractive for two reasons. First, it is a straightforward generalization of the Born-Infeld idea which involved, originally, gauge fields. Secondly, it provides a natural scale for symmetry breaking, i.e. the Planck mass. Indeed, the HGS formalism introduces a gauge symmetry which has the effect of stabilizing the Skymion but only when the large vector mass limit is taken. In other words, the symmetry must be effectively broken at all but very large scales compared to the Skymion scale. In the above treatment however, the stabilizing term is introduced through the metric via the Born-Infeld action and so it seems only natural that a large scale —of the order of the Planck scale— could be involved.

In the next section, we look at the phenomenology of the system (17). In principle it could be used in several contexts: low hadron physics, weak Skymions where the Born-Infeld action could originate from a unified theory or even qualitons. Here, we will only consider the Skymions from the point of view of hadron physics mainly the context is well defined and it is easier to compare to the results to that of the original Skyrme model.

#### IV. PHENOMENOLOGY

We use the standard semi-classical quantization technique of Adkins et al. [10] in order to derive the Skymion phenomenology based on (17). We consider spherically symmetric and static Skymion configurations. Using the hedgehog ansatz  $U = \exp[i\tau \cdot \hat{r}F(r)]$  where  $F$  is the chiral angle, we obtain the static mass

$$M_S = 4\pi \left( \frac{f_\pi}{e} \right) \int r^2 dr \left[ \frac{\sin^2 F}{r^2} + \frac{1}{2} F'^2 - c \left( (1 - K)^{\frac{1}{2}} - 1 \right) \right] \quad (18)$$

where we have made the change of variable,  $r \rightarrow a_0 r$ , where  $a_0 = \frac{ef_\pi}{\sqrt{2}}$ ,  $c = \frac{b^2}{e^2 f_\pi^4}$  (note that for  $c \rightarrow \infty$  one recovers the Skyrme model) and where

$$K = \frac{1}{c} \frac{\sin^2 F}{r^2} \left( 2F'^2 + \frac{\sin^2 F}{r^2} \right) \quad (19)$$

A few comments are in order regarding the classical stability of the Skymion due to the non-polynomial nature of this Lagrangian. First, the Derrick theorem for testing stability against scale transformation does not trivially apply here. The argument of the square root in (18) must remain positive, constraining scale transformation to scales only down to a critical value. In other words, the constraint of a real static energy —not finiteness of the static energy like in the Skyrme model— prevents the size of the Skymion from shrinking to zero. On the other hand, the static energy is always positive since  $-c((1 - K)^{\frac{1}{2}} - 1) > 0$ .

Minimizing  $M_S$  with respect to  $F$ , we obtain the Euler-Lagrange equation

$$\begin{aligned}
0 = & [-2 \sin F \cos F + 2rF' + r^2 F''] \\
& + (1 - K)^{-\frac{1}{2}} \left[ 2 \sin F \cos F \left( F'^2 - \frac{\sin^2 F}{r^2} \right) + 2 \sin^2 F F'' \right] \\
& + \frac{1}{c} (1 - K)^{-\frac{3}{2}} \left[ 4 \frac{\sin^2 F F'}{r} \left( \left( \frac{F'}{r} \sin F \cos F - \frac{\sin^2 F}{r^2} \right) \right. \right. \\
& \quad \left. \left. \left( F'^2 + \frac{\sin^2 F}{r^2} \right) + \frac{F'}{r} \sin^2 F F'' \right) \right]
\end{aligned} \tag{20}$$

As in the case of the standard Skyrme equation, (20) admits no exact analytical solution and has to be solved numerically. Under rotation, we find that the Skyrmion inertia is given by

$$\mathcal{I} = \frac{8\pi}{3e^3 f_\pi} \int r^2 dr \left[ 1 + (1 - K)^{-\frac{1}{2}} \left( \frac{\sin^2 F}{r^2} + F'^2 \right) \right] \tag{21}$$

In computing the preceding expression we have neglected terms which are quartic (or higher) in time derivatives. This approximation is only justified under the assumption that the Skyrmion rotates slowly in the semi-classical limit.

Similarly, we find the axial coupling constant to be given by

$$g_A = -\frac{8\pi}{3e^2} \int r^2 dr \left[ \frac{1}{2} F' + \frac{\sin F \cos F}{r} \left[ 1 + (1 - K)^{-\frac{1}{2}} \left( \frac{\sin^2 F}{r^2} + F'^2 \right) \right] + (1 - K)^{-\frac{1}{2}} \frac{\sin^2 F}{r^2} F' \right] \tag{22}$$

In the numerical calculations we carried out a power series expansion of the above expressions and we truncated those series to a finite order. The original Skyrme model corresponds to a truncation at fourth order, or equivalently to the limit  $c = \infty$ . The experimental masses of the nucleon ( $M_N = 939$  MeV) and of the  $\Delta$  ( $M_\Delta = 1232$  MeV) were used as input to determine the  $f_\pi$  and  $e$  parameters. On the other hand,  $c$  was left as a free parameter.

The numerical results were computed:

- (i) For different values of  $c$ , with the order of truncation fixed at 24 (Table I)
- (ii) For truncation at different orders, with the parameter  $c$  fixed at 1 (Table II).

## V. DISCUSSION AND CONCLUSIONS

From Tables I and II, we immediately see that our numerical results seem to converge toward the experimental data in the  $c \rightarrow 0$  and infinite-order limits. As infinite-order truncation corresponds to the exact solution of (20), it would seem preferable to solve the latter equation directly, for an arbitrary value of  $c$  and without resorting to a power series expansion. However we have found this alternative to be impracticable. Indeed, the numerical solution requires a finer tuning of the boundary conditions (*i.e.* the slopes of the profile at  $r = 0$  and  $r = \infty$ ) as higher orders are added. The reason for this behavior is that the solution for small distance saturates  $K$  near  $K \cong 1$  for a large range of values  $0 \leq r \leq r_0$  in

order for the series to converge in which case, the chiral angle behavior is very close to that of an  $\arcsin(ar)$ . This numerical instability has also been noted when the above-mentioned  $c \rightarrow 0$  and infinite-order limits. They have also been observed in several other infinite-order models and are fairly well understood [11] .

Looking at Tables I and II, we note that the results show very little variation in the latter limits, and thus our approximation to the exact solution seems to be working rather well. Although minimal in the case of some observables (more specifically the magnetic moments and the mean radii), the improvement achieved by our model with respect to the original Skyrme model is significant. For instance, the results for  $f_\pi$  are in much better agreement with the experimental value. We obtain a  $\sim 10\%$  accuracy with  $c = 0.01$ , as compared to the  $\sim 30\%$  accuracy for the Skyrme model. The results for  $g_A$  are also better, although they are still  $\sim 30\%$  away from the experimental value. This was to be expected, as all previous Skyrme-like models [11] showed the same feature. This gap is most probably not related to the particular choice of model, but rather to a more fundamental flaw in the approach (quantification technique, ...).

In Fig. 1 and Fig. 2, we show the behavior of the chiral angle  $F(r)$  with the same choice of parameters as in tables I and II, whereas in Fig. 3 and Fig. 4, we show the behavior of  $K(r)$  as defined in (19). We see from Fig. 1 the behavior of the chiral angle to become increasingly smoother in the low- $c$  limit. On the other hand, Figs. 3 and 4 show that the  $K$  term exhibit a plateau for small radial distances — the plateau extends further for small values of  $c$ . Another characteristic we can extract from Fig. 4 is that the  $K \rightarrow 1$  behavior of the solution is only fully expanded for high orders ( $\geq 60$ ).

To conclude, we have considered the general covariant action suggested by Born and Infeld as an alternative to the usual stabilizing term in the context of the Skyrme model. The Skymion has therefore the attractive feature of having a geometric origin. Moreover, although the results we have obtained show only a slight improvement in general in the description of the static properties of the nucleons, they are in better agreement with the experimental data than the original Skyrme model.

## VI. ACKNOWLEDGMENTS

The authors would like to thank C. Bachas for many helpful discussions and for getting us interested in the Born-Infeld action. L.M. is grateful for the hospitality of the Service de Physique Théorique (CEA-Saclay), where part of this work was done. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada, by the Fonds pour la Formation de Chercheurs et l'Aide à la Recherche du Québec and by the Fondation A.F.D.U. Québec (G.S.).



## REFERENCES

- [1] T.H.R. Skyrme, Proc. R. Soc. London **A2603**, 127 (1961).
- [2] M. Bando, T. Kugo, K. Yamawaki and T. Yanagida, Phys. Rev. Lett. **54**, 1215 (1985).
- [3] A. Dobado and M.J. Herrero, Nucl. Phys. **B319**, 491 (1989).
- [4] M. Born and L. Infeld, Proc. R. Soc. London **A144**, 425 (1934).
- [5] N.K. Pak and H.C. Tze, Annals of Physics **117**, 164 (1979).
- [6] P.A.M. Dirac, Rev. Mod. Phys. **34**, 592 (1962); S. Deser and L.K. Morrison, J. Math. Phys. **14**, 596 (1970).
- [7] J. Plebanski, Lectures on Non-Linear Electrodynamics, NORDITA-Lecture Notes (1968).
- [8] T. Hagiwara, J. Phys. **A14**, 3059 (1981).
- [9] R. Roskies, Phys. Rev. **D15**, 1722 (1977).
- [10] G.S. Adkins, C.R. Nappi and E. Witten, Nucl. Phys. **B228**, 552 (1983)
- [11] S. Dubé and L. Marleau, Phys. Rev. **D41**, 1606 (1990);  
L. Marleau, Phys. Rev. **D43**, 885 (1991);  
A.D. Jackson, C. Weiss and A. Wirzba, Nucl. Phys. **A529**, 741 (1991);  
K. Gustafsson and D.O. Riska, Nucl. Phys. **A571**, 645 (1994).

# TABLES

TABLE I. Numerical results obtained for some physical observables for different values of the parameter  $c$ , defined in equation (10). The  $c=\infty$  corresponds to the standard Skyrme Lagrangian.

	$c=\infty$	$c=1$	$c=0$	$c=0$	$c=0$	$c=0$	Expt
$f_\pi$ (MeV)	64.6	76.8	78.2	80.8	81.6	82.8	93
$10^3 \epsilon^2$	4.22	1.63	1.31	0.74	0.57	0.29	
$\langle r_E^2 \rangle^{1/2}$ (fm)	0.59	0.61	0.61	0.62	0.62	0.62	0.72
$\langle r_M^2 \rangle^{1/2}$ (fm)	0.92	0.86	0.86	0.85	0.85	0.85	0.81
$\mu_p(\mu_N)$	1.87	1.89	1.90	1.90	1.90	1.91	2.79
$\mu_n(\mu_N)$	-1.33	-1.31	-1.31	-1.30	-1.30	-1.30	-1.91
$g_A$	0.61	0.78	0.80	0.85	0.86	0.89	1.23

TABLE II. Numerical results obtained for some physical observables for some different orders of truncation. The 4<sup>th</sup> order corresponds to the standard Skyrme Lagrangian.

Order	4	8	12	24	60	80
$f_\pi$ (MeV)	64.6	73.6	75.4	76.8	77.3	77.4
$10^3 \epsilon^2$	4.22	1.80	1.63	1.58	1.58	
$\langle r_E^2 \rangle^{1/2}$ (fm)	0.59	0.60	0.61	0.61	0.61	0.61
$\langle r_M^2 \rangle^{1/2}$ (fm)	0.92	0.88	0.87	0.86	0.86	0.86
$\mu_p(\mu_N)$	1.87	1.89	1.89	1.89	1.89	1.89
$\mu_n(\mu_N)$	-1.33	-1.32	-1.31	-1.31	-1.31	-1.31
$g_A$	0.61	0.73	0.76	0.78	0.79	0.79

## FIGURES

FIG. 1. The chiral angle solution to (20) function of the radial distance (in units of  $a_0$ ) for values 1, 0.5, 0.1, 0.05, 0.01, 0.005 of the parameter  $c$  defined in (18,19) at order 24. The lowest curve in the graph corresponds to  $c = 1$  and the highest one to  $c = 0.005$ .

FIG. 2. The chiral angle solution to (20) function of the radial distance (in units of  $a_0$ ) for orders 8, 12, 24, 40, 80 of truncation with  $c = 1$ . The lowest curve in the graph corresponds to 8<sup>th</sup> order truncation and the highest one to 80<sup>th</sup> order truncation.

FIG. 3.  $K(r)$  defined in (19) function of the radial distance (in units of  $a_0$ ) for values 1, 0.5, 0.1, 0.05, 0.01, 0.005 of the parameter  $c$  defined in (18,19) at order 24. The lowest curve in the graph corresponds to  $c = 1$  and the highest one to  $c = 0.005$ .

FIG. 4.  $K(r)$  defined in (19) function of the radial distance (in units of  $a_0$ ) for orders 8, 12, 24, 40, 80 of truncation with  $c = 1$ . The lowest curve in the graph corresponds to 8<sup>th</sup> order truncation and the highest one to 80<sup>th</sup> order truncation.

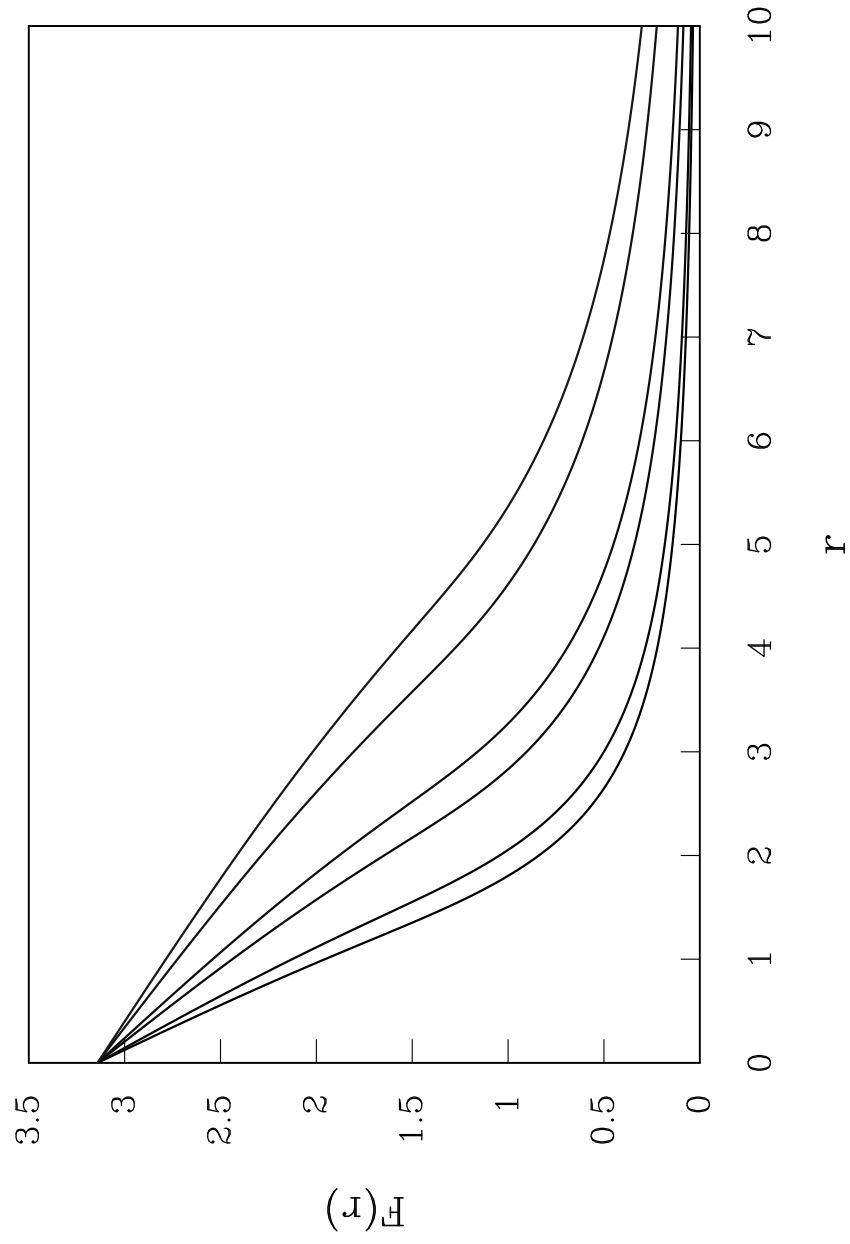


fig.1

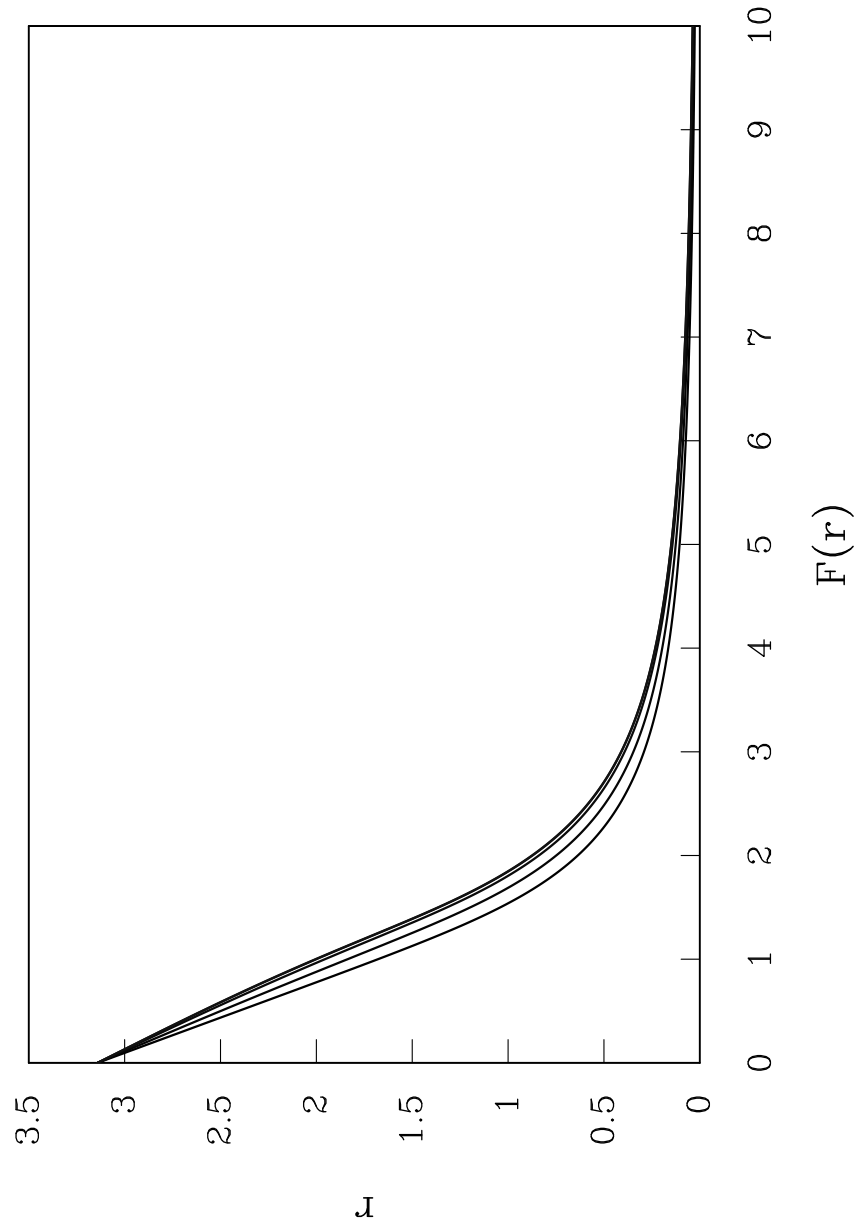


fig.2

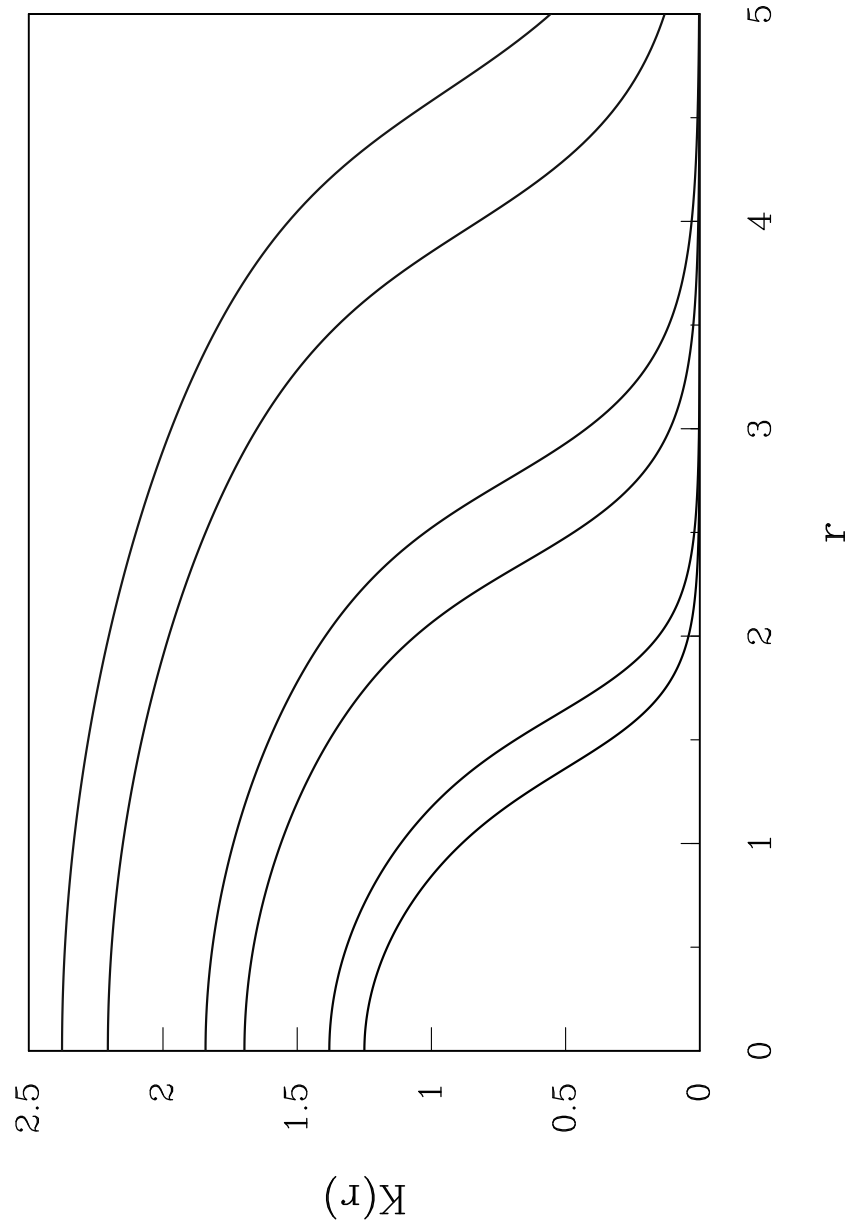


fig.3

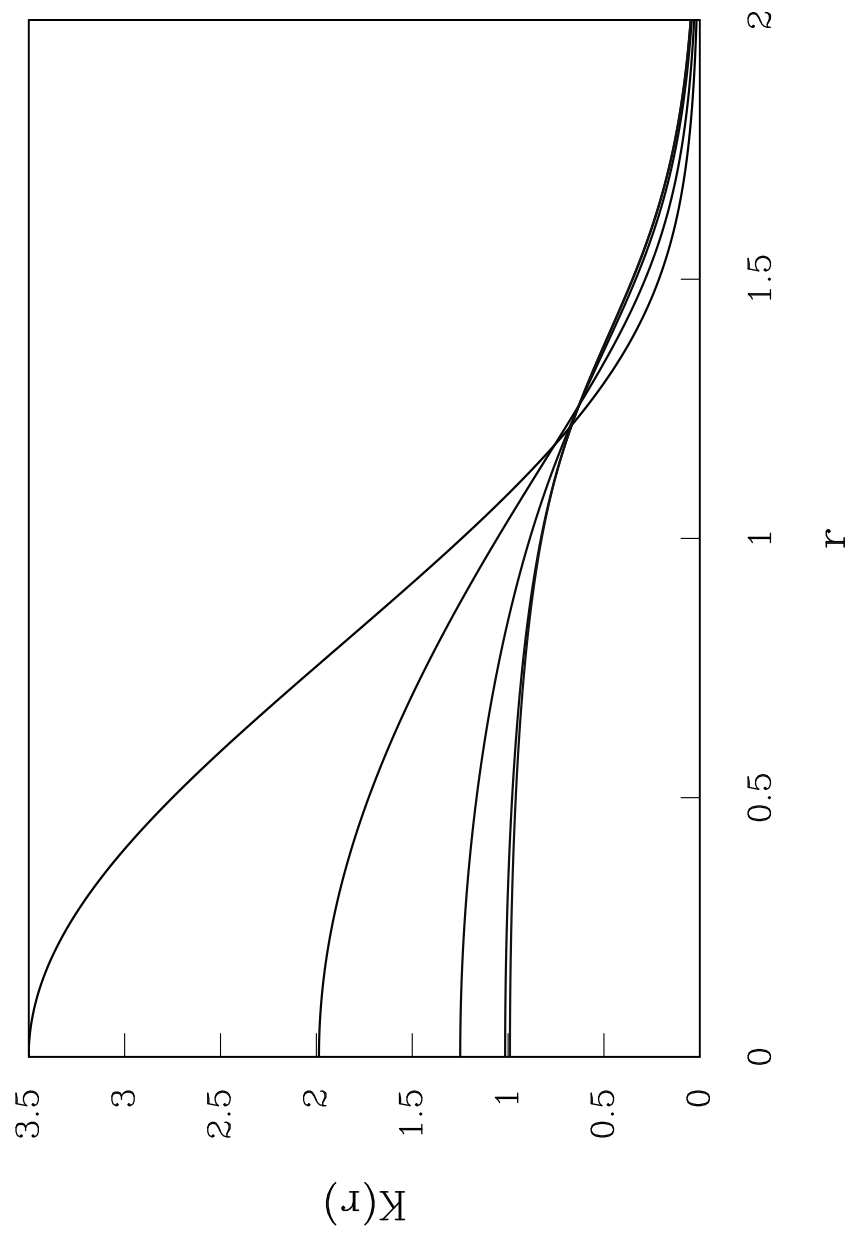


fig.4